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The Stratified Learning Zone: Examination of the Pros and Woes of Collaborative-Learning Design in Demographically-Diverse Mathematics Classrooms

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A *stratified learning zone* is a design-engendered hierarchy of student learning trajectories, each delimited in its conceptual scope, and all simultaneously occurring within a classroom. This paper examines the emergence of a stratified learning zone as a result of student interaction during an implementation of a combinatorics collaborative construction activity in 6th-grade classrooms. Within groups, students self-organized into assembly-line roles according to personal interest, mathematical inclination, and student-to-student negotiation and coordination. The *classroom* completed the project, but *students* differed in the learning opportunities they experienced. We critique the equity of such design, particularly in demographically heterogeneous populations. Students are not optimally challenged, because tacit team forces inculcate student roles, thus precluding all students' exposure to the mathematical concepts underlying the design. We portray the gradual evolution of student roles in order to explore and address design tradeoffs inherent in collaborative-learning projects. We present teacher solutions for tempering the stratification and suggest networked-classroom design as potentially affording more equity and inclusion.

Introduction

Classroom projects can be effective 'kick-off' activities for constructivist curricular units (Edelson, Pea, & Gomez, 1996; Kolodner et al., 2003; Wilensky & Stroup, 1999a). These kick-off activities often involve students' collaborative construction of classroom artifacts, such as models, computer programs, or presentations. Such 'constructionist' activities (Papert, 1991) are designed so as to foster intrinsic motivation— in order to successfully complete the construction of the artifacts, students are to engage in problem solving that bootstraps key ideas of the curricular unit. Construction projects may involve planning as well as execution, because on the one hand, students are not given blueprints to guide their construction, and on the other hand, material, human, and time resources are limited. In fact, ambitious projects may demand detailed planning and arduous execution. Students come to realize that they are co-dependent for the success of the project; that they cannot complete the project individually. Students' awareness of their co-dependency can potentially engender a social negotiation and distribution of design and construction tasks—the project becomes a multifaceted concerted effort. Ideally, emergent issues of management and specialization become intertwined with the core curricular issues at stake in the problem-solving process, so that activity in the social forum grounds a growing understanding of content. For instance, if students are creating a poster of types of animals, then by virtue of distributing the work into task forces corresponding to mammals, amphibians, fish, etc., students are already engaging in an essential part of the taxonomy in question.

The emergent social dynamics of collaborative construction projects may merit careful study, especially in student populations where social issues are at stake. This paper looks at a collaborative construction activity in a mathematics classroom of high demographic heterogeneity and focuses on the distribution of tasks between students. We will examine whether this distribution is equitable in terms of student learning opportunities it affords and whether any possible inequity that we may find should inform the design and implementation of collaborative construction activities. Specifically, collaborative construction demands of the classroom a variety of skills ranging from design and engineering through production and supervision, but what can we say of a classroom that successfully completed a challenging construction project? Did the 'classroom' learn? What exactly would that mean? Is 'distributed expertise' a sufficient and desirable goal of design for school mathematics (as compared, say, to a workplace)? Do many if any students get the whole picture? It could be that students choose to narrow their participation to activities they identify with, feel adequately competent in contributing towards, and are consistently credited for. Moreover, in labor-intensive projects, the less mathematically competent students may find themselves typecast as suitable for jobs that are lower on the mathematical ranks. No individual malice would be involved in perpetuating certain students' mathematical labeling—such perpetuation would arise as an emergent phenomenon of the classroom system: students would be rewarded for their manual labor, because the labor is necessary for the classroom's overall success; the reward would encourage these students to persevere in the manual-labor roles; and the facilitator who witnesses an *overall* 'on-schedule' progress of the project may communicate a positive sanctioning of the labor distribution by "mathematical caste." Such typecasting is liable to carry pernicious social baggage when it reflects students' socio-economic background and ethnicity, so that an implementation of a collaborative project, which at first glance appears to be a constructionist idyll, is in fact a Huxleyian nightmare.

The objective of this paper is to expose the potential inequity underlying students' industriousness in collaborative construction projects. Designers and facilitators may be informed by this study. First, the phenomenon may be unfamiliar to many practitioners, and so this paper will sensitize them to its existence and liabilities. Secondly, we will analyze the tacit rules of social negotiation that give rise to the phenomenon. Thirdly, we will suggest several design principles for tempering the mathematical typecasting—a tempering that may possibly come at the short-term expense of project progress but may ultimately engender long-term learning benefits for more students (see Axelrod & Cohen, 1999, on *exploration vs. exploitation*).

The remainder of this section introduces and explains the design of the combinations-tower activity that constitutes the backdrop of this study.

Data Source
This paper analyzes socio-mathematical dynamics in two joint implementations of the *combinations tower* design (see below). The implementations were conducted as part of our design-based research of constructivist learning-environments for mathematics (e.g., Abrahamson & Wilensky, 2004a, 2004b, 2005a).

Design of the Collaborative Construction Project
The combinations tower is the introductory activity of *ProbLab* (Abrahamson & Wilensky, 2002, 2005b), a probability-and-statistics experimental unit under the auspices of the *Connected Probability* project (Wilensky, 1997). In ProbLab, students interact with tools and create artifacts

both in traditional and computer-based media (see Abrahamson, Blikstein, Lamberty, & Wilensky, 2005, on *mixed-media learning environments*). One objective of the ProbLab unit is that students will understand probability as a coordination between theoretical work and empirical experiments. The theoretical-probability work consists of analyzing, determining, and literally constructing combinatorial spaces of stochastic objects. The complementary empirical-probability work consists of conducting computer-based simulation of probability experiments with these same stochastic objects. The ProbLab learning tools furnish facilitation infrastructure supporting classroom discussion through which students are to compare between the theoretical- and empirical-probability artifacts. These artifacts, as we now explain, are designed to appear perceptually similar.

The combinations tower is a concrete-media activity that constitutes a reference model during subsequent virtual-media activities. Just as many traditional designs for probability incorporate flipping coins and rolling dice, our computer-based counterparts to these mechanical experiments incorporate the *9-block*. The 9-block is a 3-by-3 grid, in which each of the nine squares can be either of two colors, such as green or blue (see Figure 1, below). In the virtual activities, implemented in the NetLogo environment (Wilensky, 1999), the coloration of 9-blocks is governed by computer-generated randomness, just as flipping coins or rolling dice are governed by physical mechanisms. And just as students know what a coin or die might land on, so students need to experience the different possible outcomes of a randomly generated 9-block so as to anticipate and interpret outcomes of probability experiments. Thus, an introductory activity is for students to explore and construct the entire combinatorial space of 9-blocks—what the 9-blocks might "land on"—and to progressively attend to quantitative–relational aspects of this combinatorial space.

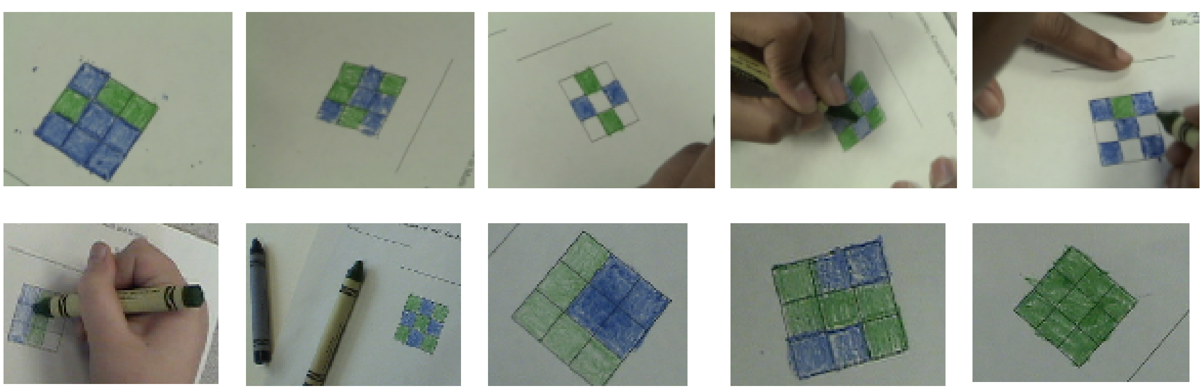


Figure 1. Students each create a single "9-block."

Students soon discover that there are many different combinations (see Figure 2, below, for student work on the second worksheet). Questions arise as to whether the number of combinations is finite, how large it could be, how students could determine this number and generate these combinations, and in particular, how students could avoid creating duplicate combinations. Thus, issues of design, engineering, and management arise as the classroom attempts to coordinate the production of the entire combinatorial space.

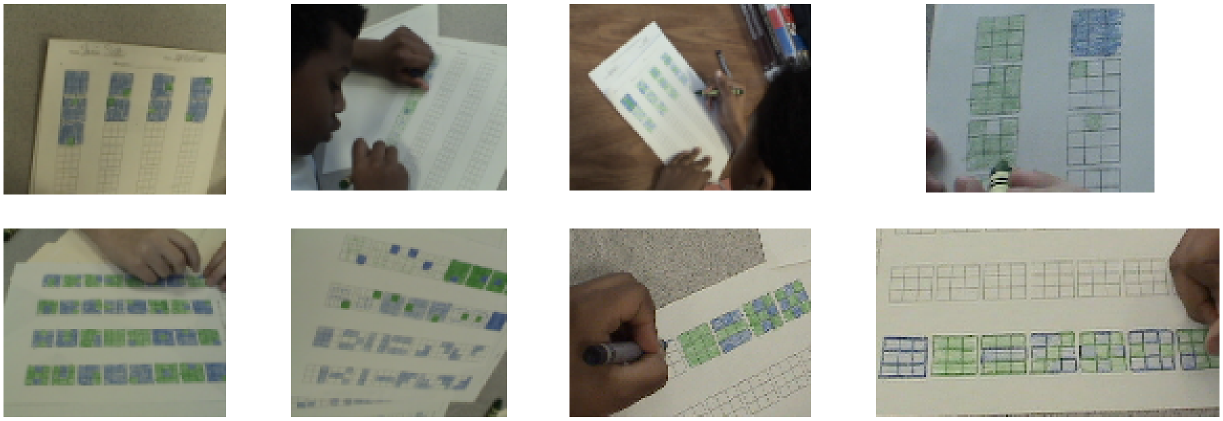
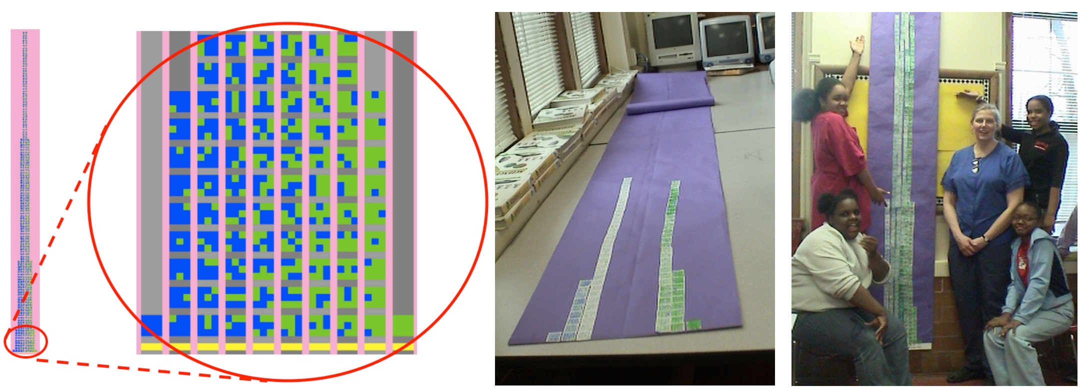


Figure 2. Students extend their combinatorics inquiry of the 9-block.

Some students stumble upon a combinatorial-analysis strategy that attends to the number of green squares within the 9-block (0, 1, 2, …9). The facilitator leverages this strategy as a basis for collaborative construction of a shared artifact that enhances the global mathematical message inherent in the combinatorial space. Specifically, the facilitator guides students to assemble the 9-blocks onto a poster in the form of a histogram according to the number of green squares in each 9-block (see Figure 3, below, for phases in the construction of the combinations tower). (The columns consist of 1, 9, 36, 84, 126, 126, 8, 36, 9, and 1 blocks—the coefficients of the binomial distribution of *(a+b)^9*.) This shape, the teacher explains, will communicate the relative magnitude of each class of 9-blocks. For instance, one can readily judge that there are many more 9-blocks with 4 or 5 green squares (126 9-blocks in each) as compared to those with 3 or 6 green squares (84 9-blocks in each), etc. (note, in Figure 3, below, that the columns grow towards the middle of the tower). This histogram structure constitutes an organizing scheme—students can more readily monitor the 9-blocks that they have already created and have a model for distributing the remaining construction amongst the classroom—but such organization, in turn, introduces new engineering and management challenges.



a. *b.* *c.*

Figure 3. The combinations tower: (a) a computer simulation of the combinations tower, with the complete tower on left and an enlargement on its right; (b) a partially assembled combinations tower; and (c) the completed combinations tower extending up to the ceiling.

Note that this paper focuses on classroom socio-mathematical issues of design and learning. That is, even though it is *we* who designed the activities and learning tools, we will mostly treat the design as a given that contextualized the issues at hand. Similarly, we will dwell on student mathematical inventions and solution methods only to the extent that these help contextualize the research questions. For a broader discussion of the design problem and student ideas, see Abrahamson and Wilensky (2005b), and for a cognitive perspective on student learning in the design, see Abrahamson and Wilensky (2005c).

Methodology

Participants

A total of 40 students in two 6^{th}-grade classrooms (the "AM" or morning classroom and the "PM" or afternoon classroom) participated in a two-day[1] (80 minutes per day) implementation of the combinations-tower design in a middle school in a very heterogeneous urban/suburban district (school demographics: 43% White; 37% African–American; 17% Hispanic; 2% Asian; 36% free/reduced lunch; 5% ESL). The teacher was a White female mathematics-and-science teacher in her second year as a teacher. The first author and the classroom teacher co-planned some of the material and logistical aspects of the implementation (primarily measuring the classroom height, calculating the necessary size of each 9-block, and producing the worksheets) and shared the facilitation of the activities. In this teacher's classroom, students sit in groups with either four or five students per group.

Collected Data

Our data include a total of about 5.5 hours of video footage from the implementation of the design. The "free-range" nature of classroom activity enabled the first author to circulate with the video camera among student groups and elicit student descriptions both of the tasks they were engaged in and of the implicit social negotiation that led them to engage in these tasks. Also, we have student work from post-test questionnaires that evaluated student ability to apply what they had learned to different numerical cases. In order to elicit the teacher's perspective on the distribution of tasks in her classroom, we interviewed her after the intervention. The interview, which lasted 45 minutes, was videotaped.

Data Analysis

We coded participant students according to three scales: (a) mathematical achievement; (b) SES; and (c) project role. To determine student mathematical-achievement group, we asked the mathematics teacher to rank students according to a three-value scale ("1" is the highest). Thus, for example, a "2 student" and a "3 student" working together would constitute a "2.5 pair" (because 2.5 is the mean of 2 and 3). To obtain student SES group, we interviewed both the mathematics-and-science teacher and her colleague, a veteran African–American teacher, who teaches the same students the other 6^{th}-grade subject matters. They based their judgment on their knowledge of the education and occupation of the students' parents and on the students' housing conditions. The two teachers were unanimous with respect to their SES scoring ("A" is the highest). To determine student project role and any changes in this role, we examined the video data and marked which role each student was engaged in. Also, through examining the video data we strove to build an understanding of how student leadership and the division of labor

[1] Students continued working independently of the research for a third duration of 80 minutes.

within the classroom occurred. In particular, we searched for the locus of initiative for leadership and division of labor and any student-to-student and student-to-teacher interactions that may have impacted these phenomena. Finally, we scored students' post-test work on an item that was designed to reveal student understanding of the combinatorial-analysis methods used in the unit. We ranked students from the most complete performance to the least complete performance, allowing for ties when we could not determine or agree on rank. This list was then divided into three distinct performance groups roughly equal in size: "complete," "partially complete," and "poor."[2]

The teacher post-intervention interview was fully transcribed and shared among members of the research group, who marked and commented on passages in this document that they found relevant to the study. These commented passages enriched our interpretation of the video data; the passages drew our attention to particular episodes, illuminated teacher actions, and became the basis for a discussion around the teacher's pedagogical beliefs and practices and any apparent discrepancies between the reported practices and the actual practices as seen in the data.

Results and Discussion

The combinations-tower design was implemented in two 6th-grade classrooms. Broadly, these parallel implementations were very similar, with some facilitation lessons learned from the "AM classroom" that were immediately applied in the "PM classroom." As we explain below, the parallel implementations actually met, because the collaborative project was too large for a single classroom to complete independently, and so students from the two classrooms coordinated their work on the combinations tower. In discussing the socio-mathematical dynamics that emerged in these implementations, we will treat the two classrooms as one and highlight only any interesting differences between the classrooms.

This section follows the implementation, focusing on enforced and emergent properties of student role within and between groups. We explain connections between management tasks, mathematical content and cognition, and classroom social dynamics. Next, using complexity-studies and socio-cultural perspectives, we examine the potential of the designed and constructed artifacts in enabling a sharing but also a compartmentalization of knowledge. We then present findings from student response on a post-test item, followed by the teacher's reflections on the implementation and, specifically, her role and students' roles in the distribution of tasks between and within groups. The paper ends with implications of this study for design and teaching, particularly in light of the unique demographics of the school district.

Initial Classroom Dynamics

This sub-section follows the transition from individual work to group work. We show how the nature of the activity gave rise to spontaneous collaboration and then to students assuming leadership. We also discuss how the construction project became distributed between groups and how this student distribution reflected the mathematical distribution underlying the design.

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[2] Whereas there were some variations between the two parallel classroom implementations, in this paper we generally collapse this variation unless it is cogent to the study.

Spontaneous collaboration. Initially, during the first 30 minutes of the first lesson, the activity was facilitated as individual work—students, seated in their usual assigned seats, each worked on sheets containing blank 9-block grids. This activity alternated with classroom discussions of findings and strategies. No explicit direction was given yet regarding any collaborative project or any project at all, for that matter, and there was little if any sharing of ideas within and/or between tables. Students varied in their type of comments, and these comments were later reflected in the tasks students chose to engage in, once the collaborative project had been announced and students began moving within the classroom space and regrouping. For instance, one student who, during individual work time devised a mathematical formula for determining the total number of different combinations of the 9-block, would later congregate with other "number crunchers" (see below); another student who searched for a strategy for creating different patterns would then work with like-minded "designers" (see below).

About half an hour into the first lesson, as the enormousness of the task began dawning on some of the students, and even before the project had been announced, collaboration sprouted spontaneously. For example, when Jenny said to the first author, who was interviewing her on-the-fly, "This is going to take *way* too long," Ki-Wee, seated at the next table, cried out, "I can do a page, Jenny!"[3] Quarter of an hour later, once the collaborative project had been announced (creating all the different combinations of the 9-block), students began devising strategies for organizing the collaboration, and the apparently more effective strategies were chosen. The students whose strategy was chosen were asked to help facilitate the strategy.

Assuming leadership. The first implementation day had been on a Tuesday. On the second implementation day, a Thursday, Jenny came to the blackboard and taught the classroom the *"anchor–mover"* combinatorial-analysis method she had devised together with Milly and Sunny, two students from the other classroom. These girls had all stayed on after class on the previous implementation day to work further on the mathematical problem. They had also met independently on Wednesday to try and create general solutions to the problem in the form of equations. Milly and Sunny, afternoon students, were present in the morning lesson—they had been invited by the teacher to help with the project. In the anchor–mover system, successive 9-blocks with *n* green squares are produced in a rigorous method geared to exhaust all the exemplars without repeats (see next section for further explanation). These students also discovered the rudiments of a system for determining the number of different *k*-subsets out of *n* items, albeit they were still struggling to articulate this discovery as a general formula. Answering a student's question as to whether the anchor–move system applies to the general case, Jenny said that

> We figured out, like, an equation, but we thought the equation would be, like, kind of confusing, because *we* were kind of confused by it. So we're trying to find, like, a way that we can just make it, like…like more of…more of, like, a… Like, we're try… You…like, you can use a shortcut to find all the combinations, but we thought it would be better if we just made it go the long way, because <u>that's a lot easier to understand even if it takes more time</u>.

[3] All student names have been replaced with pseudonyms.

In uttering the last clause (see, above, the underlined words), Jenny, perhaps inadvertently, put her finger on a pivotal element of our design rationale for the combinations-tower activity. A small number of students, high mathematical achievers, could potentially appropriate the anchor–mover algorithm just from watching Jenny's gestures on a single empty 9-block grid she had drawn on the board. That is, the mathematical knowledge Jenny wishes to communicate can be seen as embodied in a procedure, and if a student has fully understood the procedure, this student may not need to apply it (to "run" it). However, most students need to engage in enacting this algorithm ("running the procedure"). Only thus would these students understand the rigor and systematcity of combinatorial analysis and appreciate the proportional relationships between meaningful groups of items within this space. A context for practicing the algorithm is in actually building each and every one of the possible 9-blocks, and the combinations tower is a potentially engaging project that focuses and organizes student work as well as bridges to probability experiments.

Jenny, Milly, and Sunny, in coordination with the teacher, had divided the classroom into equally-sized task-specific groups that would work, respectively, on creating 9-blocks with zero and one green squares, with 2 green squares, with 3 green squares, and with 4 green squares. The plan was for the afternoon classroom to create the rest of the blocks (with 5, 6, 7, 8, and 9 green squares, i.e. with 4, 3, 2, 1, and 0 blue squares). The mean level of students' mathematical-achievement by group is as follows (1 is highest): Group #1 is 2.4, Group #2 is 3.0, Group #3 is 1.6, and Group #4 is 1.2. Although the mean mathematical level does not increase monotonically, the teacher reported that this distribution of work between the groups was informed by student composition, with the stronger groups being assigned the more demanding construction of the larger classes of 9-blocks (mean of 1.4 as compared to 2.7). This distribution was more marked in the afternoon class, where students were assigned into only three groups who were in charge of building the 0-, 1-, and 2-blue blocks (Group #1), the 3-blue blocks (Group #2), and the 4-blue blocks (Group #3). Group #1 was composed of 5 students with a mean mathematical achievement score of 2.4. Group #2 had 6 students, with a mean of 2.0. Group #3 had 7 students, with a mean of 1.1.

Once Jenny, Milly, Sunny (the "coordinators") and the teacher had assigned students to their groups, students regrouped and began working on their respective tasks. Thus, 20 minutes into the second day, the classroom began a concerted effort to produce the combinations tower. All at once, the dynamics shifted: students, huddled in their teams, became animated and local leadership and roles began to emerge. Note that the nature of the task in and of itself did not change—the expected "deliverable" was still the combinatorial space of all possible 9-blocks, as on the previous day. It is at this point in the implementation that issues of mathematical identity came to the surface, because the unit of labor was now the group and not individual students as in the earlier exercises. Thus, students each had more choice and flexibility regarding the role they enacted within the group, and student contribution was affected more by group dynamics than by teacher supervision.

The three girls were not assigned to groups, as they were reserving for themselves the roles of "coordinators" (see below; so, together with the teacher, there were a total of four classroom helpers, i.e. one for each group). Groups who completed their assigned work were either dispersed to strengthen the lines of other groups or were assigned a new task. For instance, 40 minutes into the second day, three students from the "0 green, 1 green" group were asked to begin assembling the produced 9-blocks on the poster sheet (they became "assemblers," see below).

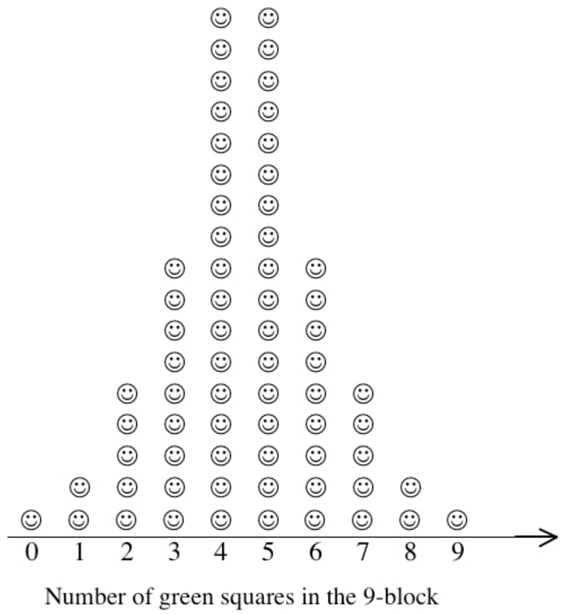

Figure 4. "Histogrammed classroom": more students were needed to create central columns of the combinations tower

Students and 9-blocks are similarly distributed. A socio-mathematical pattern emerged that related the classroom (student space) to the focal mathematical object (combinatorial space). The more 9-blocks in a column, the more students, labor, and time were needed to create them—the classroom was "plotted" onto the distribution of 9-blocks; the *classroom* was histogramed (see Figure 4, above)![4] Also, this distribution reflected student mathematical achievement—the below-average achievers could cope with the relatively simple demands of the outer columns, but for taller columns, more mathematical sophistication was called for. Thus, typically, the further into the histogram a column was located: (a) the higher the mathematical achievement was of the students who created it; and (b) the later into the implementation it was created.

Division of Labor, Coordination, and Equity

Group management and mathematical modeling. We have noted that the student leaders distributed the combinatorial space proportionately and sensitively between groups. Also within groups, student discussion suggests that students were benefiting from the constraint of sharing the production of their part of the combinatorial space. For instance, when Lot and Ki-Wee attempted to divide the production of all 9-blocks with exactly 4 green squares, Lot suddenly realized that his brute-force method was ultimately limited and, thus, he reevaluated his method. Specifically, he had been moving blocks of three adjacent green squares within the 9-block, and for each of these he placed the fourth green square in each of the six remaining grid squares. Only when was asked to administer the work within his group did it dawn on him that the three

[4] The "student histogram" is not drawn to scale. Real column heights were 1, 9, 36, 84, 126, 126, 84, 36, 9, and 1. Also, the real number of students who built the combinations tower was 40.

squares need not be adjacent; that there were many more patterns than he had expected. Group management calls for planning, which in turns necessitates a macro view of a process. Students engaging this macro view shortcut the more "groping" experimentation by articulating templates of action, which, in turn, call for mathematical language that is suitable for describing more general cases (see also Eizenberg & Zaslavsky, 2003, on how collaboration enhances control processes that positively affect combinatorics problem solving).

Between-groups. A feature of the combinations-tower construction activity is that some columns are easier to create as compared to other columns. This differential demand of the columns afforded the teacher a template by which to distribute students according to the difficulty she anticipated they would each face in completing the columns. Yet this distribution sometimes resulted in students either being underchallenged or overchallenged. That is, Group #1 students, who were assigned to build the short outer columns, did not face *strategic* challenges, because by that point the anchor–mover strategy had been established and disseminated. Many students in other groups were more overchallenged, because at that point the classroom had not yet discovered how to apply the anchor-mover strategy recursively to the case of a 9-block with 3 or more green squares. To do so, these students would have had first to fully understand and practice applying the method to blocks with only 2 green squares—but they were not given such opportunity. Students who were either under- or over-challenged tended to disengage from the task, at times disrupting the group work rather than contributing towards it. Thus, whereas the enforced between-group division of labor appears sound in principle, it did not always enable students to work optimally within their zone of proximal development.

Within-groups. Within groups, following 10 minutes of socialization in their new milieus, students established working relationships. Specifically, students self-organized into a network of different roles (a "research, development, and production assembly line") that were each independent in delivery of their specific product/service, yet were dependent on other group members and other groups for periodically updated information that informed their own objectives. Note that we are distinguishing between *group* and *role.* A group is the collection of students in spatial proximity, originally assigned by the coordinators and typically seated around a cluster of desks, facing each other (albeit, some groups sprouted spontaneously around certain artifacts, as we discuss below). A *role* is associated with the specific type of task that individual students took upon themselves to engage in. Within a group, students took on different roles, yet some groups or sub-groups emerged around a common role. So some groups included students engaged in a variety of project-specific roles, whereas other groups were homogenous with respect to student roles.

The within-group production of the combinations tower was, at times, reminiscent of an ant colony that has no leader yet self-organizes in foraging food.[5] For instance, whereas some students elected to engage in computation of the combinatorial space, using induction, deduction, recursive reasoning, and visual-configuration strategies, other students preferred to draw the 9-blocks based on these strategies, search for and eliminate duplicates, or cut and paste the

[5] An ant colony is an example often used in complexity studies to describe a system of agents whose individual "local" behaviors and interactions give rise to "global" behavior that appears to be governed by a central mind. See, for example, ccl.northwestern.edu/netlogo/models/Ants .

produced 9-blocks onto the poster. At other times, the ant colony's work was boosted by central management. For instance, the teacher, who had witnessed the emergent roles ("implementers" and "checkers," see below) asked members of dispersed groups to take on those specific roles within active groups.

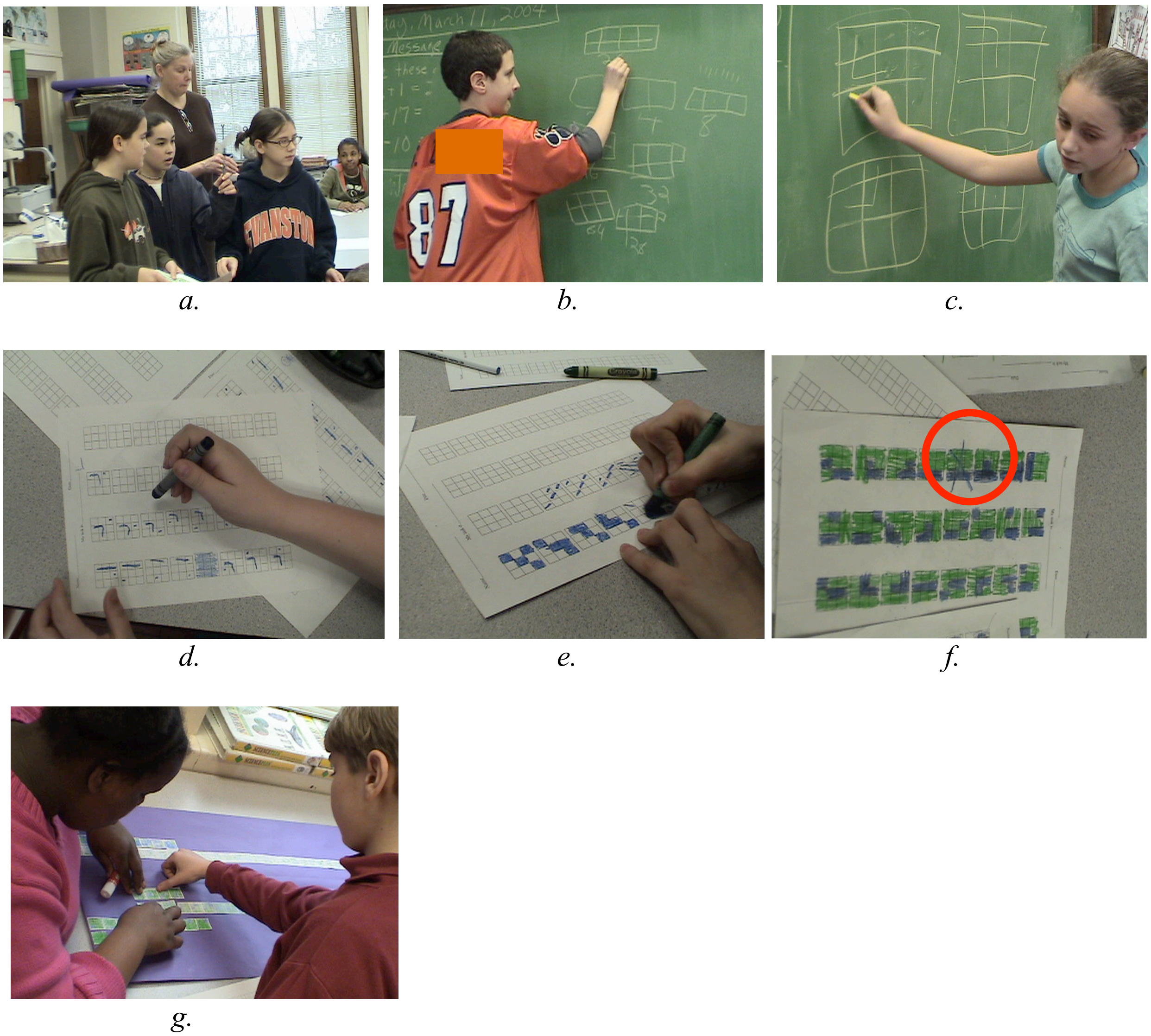

Figure 5. Identified student roles in the collaborative construction project: (a) coordinators; (b) "number crunchers"; (c) designers; (d) producers; (e) implementers; (f) checkers (note the 'X' crossing out a 9-block that duplicates a block immediately to its left); and (g) assemblers.

All in all, we identified seven roles that we have titled: (1) coordinators; (2) number crunchers; (3) designers; (4) implementers; (5) producers; (6) checkers (quality-assurance experts); and (7) assemblers (see Figure 5, above). Whereas this enumeration of roles roughly descends in terms of the mathematical reasoning these students engaged in and reflects the "assembly-line" sequence, the list does not entirely or consistently reflect an ordinal scale.[6] For instance, the "number crunchers" students and "designers" students were all engaged in high-order mathematical thinking, albeit the former students worked primarily with symbolical systems and the latter students worked primarily with visuo–spatial configurations. Through a traditional pedagogical lens, perhaps the number-crunchers were engaged in "purer" or "higher" mathematics, but such a perspective is foreign to our pedagogical practice (see, for example, Turkle & Papert, 1991). Following, we elaborate on what each of these roles consisted of.

<u>a. Coordinators</u>. The "coordinators" engaged in the most versatile role in the project. They led the classroom work on the second day (see description of Jenny, Milly, and Sunny, above; see Figure 5a, above, where the coordinators stand with the teacher). Coordinators assigned students to groups, explained the general task and the specific task of each group, and helped both disseminate practices and align them between students from different groups. Coordinators constituted content experts and performed within-group arbitration-and-management support and between-group coordination of tasks and personnel. The coordinators were openly encouraged by the facilitators to assume their role, and, thus, the facilitators delegated their authority to the coordinators, and students generally listened to the coordinators. The teacher, who appreciated the support furnished by the coordinators, asked several of them, students in the afternoon class, to join the morning class and vice versa.

<u>b. Number crunchers</u>. The "number crunchers" were students—all boys, as it was—who openly stated that they are intrigued by the mathematical problem of determining the size of the combinatorial space yet do not wish to partake in constructing the tower. Such construction, it appeared, was "beneath them," either because it appeared mathematically unsophisticated/inaccurate or because they found such hands-on tasks time consuming and just childish[7] (see Figure 5b, above, for a "number cruncher" displaying his [incorrect] guess for the size of the combinatorial space). These "number crunchers" devised a variety of mathematical algorithms but, interestingly, were never convinced of the validity of their own results. Their biggest contribution was in terms of determining the total number of items in the combinatorial space (512 items).

<u>c. Designers</u>. The "designers"—all girls, as it was—were the combinatorial analysts. They worked with the 9-block empty grids to determine fail-proof strategies for exhausting the combinatorial space. They numbered the cells in the 9-block from "1" (top-left square) to "9" (bottom-right square), and devised the *anchor–mover* system for finding all the different combinations. The 'anchor' is a square that remains green while the 'mover' switches from square to square until a sub-sequence has been exhausted, whence a new sub-sequence begins with the anchor moving one square and the mover moving through the remaining squares in the grid, and so on (see Appendix A). For instance, for the class of 9-blocks with exactly 2 green squares in them (a total of 36 blocks), the designers determined the following sequence: [1, 2], [1, 3], [1, 4], [1, 5], [1, 6], [1, 7], [1, 8], [1, 9], [2, 3], [2, 4] [2, 5], [2, 6], [2, 7], [2, 8], [2, 9], [3, 4], [3, 5], [3, 6], [3, 7], [3, 8], [3, 9], [4, 5], [4, 6], [4, 7], [4, 8], [4, 9], [5, 6], [5, 7], [5, 8], [5, 9],

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[6] The term "assembly line" is used here with the caveat that it carries no implications of mechanical behavior but is only meant to capture a type of coordinated group behavior.
[7] This interpretation is based both on student utterance and on the teacher's observations.

[6, 7], [6, 8], [6, 9], [7, 8], [7, 9], [8, 9] (a total of 36 9-blocks). This system was then applied recursively for the case of 9-blocks with exactly 3 green cells—now there was a "super anchor" in addition to the anchor and the mover: [1, 2, 3], [1, 2, 4], [1, 2, 5], …, [1, 2, 9], [1, 3, 4], …, [1, 8, 9], [2, 3, 4], …, …, [7, 8, 9], for a total of 84 different 9-blocks. For the case of 9-blocks with exactly 4 green squares, yet a "super-duper anchor" was incorporated. The entire system applied to the symmetrical classes of 9-blocks with exactly 2 blue, 3 blue, and 4 blue squares. Another different type of design involved a taxonomy of spatial configurations of squares (rows, columns, diagonals, "Ls," etc.; see Figure 5c). This latter system ultimately shipwrecked, when the students realized they could account systematically neither for all configurations nor for duplicates generated from different configurations. The designers were very much involved in teaching their algorithm to other students, whom we termed the "implementers."

<u>d. Implementers</u>. The "implementers" learned the anchor–mover system from the "designers" and proceeded to mark dots and lines into their 9-block sheets a sequence of 9-blocks (see Figure 5d, above; note that the student is only marking the squares and not filling them). These implementers gradually became skilled at the strategy, yet did not improve on it. Specifically, implementers who worked on searching for 9-blocks with 2 green squares did not figure out how to apply this system recursively to the case of 3 green blocks (but the designers did). Implementers would hand their dotted sheets to the "producers," who often sat by them.

<u>e. Producers</u>. The "producers" students received dotted sheets from their implementer colleagues. They would then crayon in the dotted squares (see Figure 5e, above), sometimes even as their implementer groupmate was working lower down on the same sheet, and finally hand the completed sheets to the "checkers," who often sat beside them.

<u>f. Checkers</u>. These "quality assurance experts" would receive from the producers sheets of crayoned-in 9-blocks. Their role was to assure that no two 9-blocks were identical. If they found such duplicates, they would cross them out and alert their colleagues (see Figure 4f, above). This role afforded primarily visual-matching activity—the checkers did not appear to follow the underlying system of the blocks—they only searched for duplicate patterns. When checkers had completed their work, the assemblers would take charge.

<u>g. Assemblers</u>. The "assemblers" were in charge of placing and securing the produced 9-blocks upon the large poster (see Figure 5g, above). They would either receive these prepared blocks form the "checkers" (the "quality-assurance experts") or they would solicit these blocks directly from the groups engaged in building them, such as when there was a lull in construction because supplies were slow in arriving. The teacher assisted the assemblers in initially outlining the columns in the combinations tower. Henceforth, the task was relatively straightforward for the outer columns but became progressively demanding for the inner columns, because the assemblers had to determine the correct orientation of the blocks.[8]

[8] In the current version of our worksheet, this would not be necessary as the blocks have a thicker upper line.

We have described seven roles, each defined by a task students specialized in.[9] Whereas such *stratification* was prevalent within groups, the stratification of roles did *not* occur only when two conditions were satisfied: (a) The students devised or were given a combinatorics-based method to distribute the work among them in terms of individual students each in charge of a specific division of 9-blocks; and (b) execution of this task was within the scope of all students in the group. This more equitable collaboration occurred in the afternoon class, in Group #1, who were working on the 0-, 1-, and 2-green blocks. For the 2-green blocks, they assigned a different anchor to each student—e.g., top left, top center, or top right, etc.—and that student had to create all the possible blocks with that anchor. It is not clear from the data whether this method for the division of labor was initiated by the group or by a coordinator. Post-test data show that students in this group, who were from the bottom or middle thirds of the classroom mathematical-achievement groups, scored either in the middle or top thirds of the classroom.

We have discussed student between-group distribution of labor and within-group distribution of roles. We now turn to examine the roles of the artifacts in student collaboration.

Collaborative Learning Around Shared Artifacts
Isolation and coordination. Some of the seven student task-specific roles involved artifacts other than the worsheets, such as the blackboard or the paper poster, and these artifacts were relatively fixed in their physical location within the classroom layout. Students filling these roles were necessarily grouped physically around these artifacts, and so they were grouped in separate locations within the classroom and, in the intensive and boisterous ecology of this design, were relatively secluded from the other students engaged in other tasks (poster students often worked in the corridor!). Thus, the classroom coalesced into islands of task-specific clusters, and these islands were each more homogeneous in terms of student mathematical achievement and role as compared to the entire classroom milieu (e.g., designers at the board, assemblers at the poster). Yet, students could also leverage the unique affordances of their respective artifacts, methods, and mathematical perspectives so as to coordinate their work, as we now explain.

Throughout the implementation a certain tension lingered within groups between those few students who did not wish to engage in actual construction and others who enjoyed doing so and possibly could not have solved the problem otherwise. Indeed, the question driving the project had been phrased as, “How many different possible patterns are there for the 9-block?,” and yet the project had then been defined as “making all of the possible patterns.” The students who focused on the research question and resisted the hands-on methodology were nevertheless not entirely isolated from their group mates—they sat at the same table as their group mates and worked with essentially the same artifact, even though they were using it differently. Thus, intermittent conversations occurred for which objectives were momentarily aligned, and these conversations were enabled by the shared artifact.

Students who made significant discoveries reported these within their groups and then, often through the coordinators, to other groups and to the facilitators. These discoveries were based in different objects and needed to be coordinated so as to advance the entire project. For instance,

[9] See also Senge & Käufer, 2000, on types of leadership—local line leaders, executive leaders, and internal networkers—that co-lead successful organizations

Samuel used paper and pen to extrapolate the total number of items in the combinatorial space (see Figure 5b, on p. 10). He achieved this by drawing a 1-block (2 combinations), followed by a 2-block (4 combinations), and then a 3-block (8 combinations), and he then continued doubling to determine that a 9-block (2^9) has 512 combinations. At the same time, Milly and Sunny had been using the sheet of blank 9-block grids to find that there are 36 9-blocks each with exactly two green squares. Lot, who had been working on a table (see Table 1, below, for the completed table), could thus enter and use these fragments of information. Specifically, students later realized that if the number of combinations totals to 512, then the unknown values in the four middle columns must total to 420, so each symmetrical pair of columns—the 3-green/4-green and the 3-blud/4-blue—totals to 210.

Table 1.
Numbers of Green and Blue Squares in the 9-Block and the Number of Different Combinations of Each in the Combinatorial Space

# Green	0	1	2	3	4	5	6	7	8	9
# Blue	9	8	7	6	5	4	3	2	1	0
# Combinations	1	9	36	*84*	*126*	*126*	*84*	36	9	1

Note: This table constituted the basis for organizing the combinations-tower histogram.
The italicized numbers are those that were discovered latest. Students had determined that these numbers totaled to 420.

This table (see Table 1, above) enabled coordination between the mathematical "purists" and the 9-block "designers"—each party could potentially appreciate the others' mathematical reasoning styles. Specifically, this was an opportunity for Samuel to realize the importance of actually constructing the blocks as a form of validation. The ensuing debate at the blackboard attracted six onlookers. When agreement could not be achieved, the table became a battleground between Samuel's logical deduction and a conjecture that had come from one of the groups. That group claimed that there are 252 combinations with 3 green squares (incorrect). They had remonstrated that Samuel's figure, 512 (correct), could not be correct, because it was not divisible by 9. Samuel was outnumbered, so to speak, and retreated to reexamine his calculations. In sum, the blackboard functioned as a shared space for mathematical reasoning—it constituted an artifact that enhanced collaborative argumentation (see Abrahamson, Berland, Shapiro, Unterman, & Wilensky, 2004). Curiously, the one tool *all* students shared—the 9-blocks worksheet—did not always afford such fruitful debate, as we now explain.

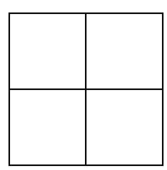
The combinations-tower project is such that once it is fragmented into sub-tasks, e.g., implementing, producing, and checking for duplicates, expertise in each of these tasks does not necessarily give students tools for reconstructing the entire task. This fragmentation and the myopia it produces can be contrasted with collaboration in a soccer team, where all players, though they are assigned specific roles associated with the ball, do have insight into their team players' roles, including how they reason and respond in the face of shifting contingencies—there is a shared culture and body of knowledge around the practice and spectatorship of the soccer event. Yet as the 9-block sheet passes down the production chain from one actor to another, it functions as a *boundary object* (Star & Griesemer, 1989; Wenger, 1998)—it takes on different nuances of meaning reflecting local practice which may have only partial overlap one with the other. At the same time, the sheet constitutes the pivot artifact coordinating the goal-oriented functioning of the classroom distributed cognition (see also Hutchins & Klausen, 1996; Pea, 1993). The 9-block grids are both the substance that is to become the body of the combinations tower and the organizing artifact that actuates and accretes individuals' labor, conflated signatures tracing knowledge from the descending strata of mathematical expertise.

Finally, what is difficult to capture in tables and scales yet is pertinent to appreciating the complexity of issues of classroom social dynamics and learning is the overall enthusiasm with which students engaged in the project. Almost all of the students were very busy during the lessons, animatedly debating their solutions and working on their respective tasks. We witnessed voluntary acts of personal investment: Many students remained in class in between lessons and after school to coordinate work between the two classrooms, and some students took work home so as to continue producing the 9-blocks for the taller columns of the combinations tower. The teacher reported that the project was difficult to accomplish yet very rewarding both for her students and for her. The combinations tower remained in the classroom as a "trophy," was later referred to as a resource in a subsequent mathematics unit, and was discussed on parent nights.

We have described emergent and controlled socio-mathematical phenomena that occurred during the implementation of the combinations-tower design. We have discussed the ambiguous functioning of designed artifacts as affording either coordination or isolation of knowledge. What is the connection between emergent-vs.-controlled learning and the potential of an artifact to serve in the negotiation and mediation of knowledge? Following a brief section on student performance on a post-test item, we will present teacher reflections that shed light on this question.

Post-Test
Following the combinations-tower project, students completed a questionnaire that included feedback questions and some content questions.

This is a 4-block. It is empty.



Make a combinations tower of all the different black-and-white 4 blocks.

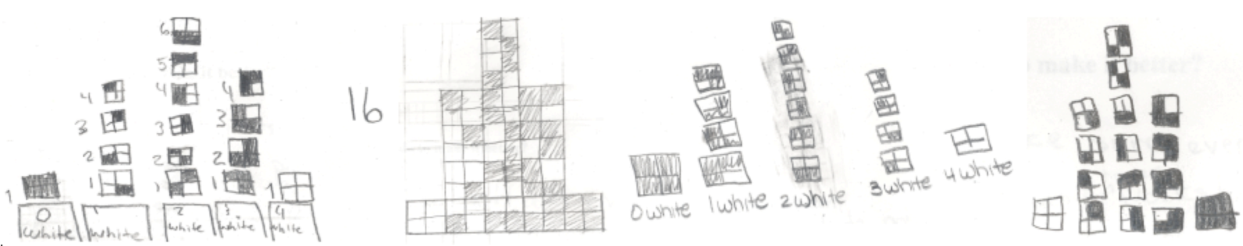


Figure 6. A post-test item (above) and four examples of student work (below): Students built combinations towers of 4-blocks (there is a total of 16 different blocks).

Results on the item (see Figure 6, above) were that: 38% of the students completed the item perfectly; another 36% either omitted a single block (most typically the all-white "empty" grid) or repeated one or two blocks; and all but three of the remaining 26% of the students (two LD and one who had been absent on the first day) manifested some elements of the structure and/or combinatorial system of the combinations tower.

Student performance on the post-test is was related to their mathematical achievement group (see Table 2, below). Note that the purpose of this study is not to evaluate whether the design equalized student performance, but whether it afforded equal opportunities for each student to progress as far as possible along their own learning path. In terms of the table (see Table 2, below), a design objective would be to eliminate column 'c,' where currently a quarter of the students reside. In terms of this paper, the analysis objective has been to understand how better to leverage the social dynamics of the classroom so as to enable the design objective.

Table 2.
Student Distribution by Mathematical Achievement Group and Post-Test Ranking

Mathematical Achievement Group	Post-Test Score		
	a*	b	c
1	9	3	-
2	4	9	2
3	2	2	8

*"a" is the highest-achieving third

We now turn to the teacher input, our interpretation of this input in terms of complex systems, and finally to implications of these insights for the design of collaborative learning environments.

Teacher Interview

In reporting the teacher's input interwoven with our interpretation of this input, we first present her observations of how student-to-student interaction played a major role in the distribution of labor. After an analysis of these interactions from a broader perspective, we present the teacher's reported methods for anticipating and responding to student-to-student interactions so as to foster as equitable a learning environment as possible. Interestingly, the properties of student-to-student interactions resonate with frameworks for the analysis of complex systems, such as social systems (e.g., Holland, 1995). Earlier, we briefly mentioned the case of an ant colony as a canonical complexity-studies example of how the local actions of multiple actors ("agents") can give rise to a system-level pattern. From this perspective, the student-to-student interactions can be regarded as *agent-based*, and the social patterns can be regarded as *emergent* (in the sense of 'not premeditated'). On the other hand, the teacher's actions are intended, often even pre-planned. These teacher-initiated interventions for precluding, monitoring, and controlling for undesirable outcomes of these emergent patterns can thus be regarded as methods for optimizing student learning under the special social conditions enabled by and arising from the looser control she exercises in her classroom. Thus, the student-based and teacher-based actions are in constant dialectic, treading a facilitation tightrope between pedagogy and equity.

An agent-based explanation of the emergence of student task distribution. The teacher's report shed new light on our analysis of the spontaneous evolution of classroom task distribution. She described how student specialization emerged as a function of individual student interactions: Within a group, once a student realized that he had reached his limit in terms of mathematical problem solving as compared to another student within that group, the first student would often capitulate to his group-mate the task of pursuing that mathematical problem, and then *she* would take over, relegating to him a necessary task that was within his zone of achievement, thus freeing her to focus on the problem he had abandoned. A network of symbiotic relationships crystallized as the more advanced students assumed leadership of their groups and as the emergent task specifications were articulated in terms of student roles and student-to-student and group-to-group negotiated partnerships. The likelihood of an individual student dominating another was affected by personality traits. Mathematically-advanced students who are typically reticent and not as socially fluent as their peers preferred to work individually, whereas "bossier" students were more likely to assign tasks to other group members—they wielded their knowledge as power.

The following week, the first author was speaking with the classroom, which had just completed building the combinations tower. Students were describing their respective roles in building the tower, including one student who said his job was to "cut and paste," another who said he was a checker, and yet others who were implementers and producers. The first author asked the classroom how they had decided who was to fill which role. After a moment of hesitation, one student, who had been a number cruncher, said, "We drew straws." Other students quipped, "No we didn't," and the first student smiled and said, "We… we just decided."

Exploration versus exploitation. When a classroom engaged in collaborative project-based activity is progressing towards successful completion of the project, could there be any justification to tamper with this progress? On the other hand, is a facilitator ethically permitted to sacrifice individual students' learning so as ensure the completion of the project? To address this design-and-facilitation dilemma, we will now turn to a complexity-studies perspective on organizations. One could arguably model the study classroom as an organization, a collective of individuals with some shared objective and a modus operandi for working towards this objective. There are no monetary stakes involved, but certain roles enable some students to gain knowledge capital, while other roles do not. The motivation to model the classroom as an organization is that a curious overlapping of tacit practices—of learning spaces and of working spaces—may not be entirely beneficial for all students.

Axelrod and Cohen (1999) discuss *exploration versus exploitation*, a tradeoff inherent in *complex adaptive systems*, such as organizations. For instance, in allocating resources, an organization must determine which strategy will maximize its benefits—"mutating" to check for better fits with the changing environment or stagnating and cashing in on a proven model of success. Typically, "the testing of new types comes at the some expense to realizing benefits of those already available" (Axelrod & Cohen, 1999, p. 43). We submit that a classroom can be seen as a complex adaptive system (Hurford, 2004), at least in terms of students' within-group free-range agency in problem solving and the interactions that shape behaviors of these agencies. Initially, all students are explorative. Yet, once a functioning coordination scheme has evolved that is apparently well adapted to the environment, i.e. the classroom-as-a-whole is apparently progressing along a trajectory towards completing a prescribed task successfully and positive sanctioning is received from the forces that be (the facilitators), an implicit quietus is set on any further exploration, and the group achieves dynamic stability. From that point on, the individual cogs of the production mechanism hone their skills and produce.

Group-level intervention for overcoming mathematical segregation. The teacher often enacts group-based learning projects both in science and mathematics. She tries to seat struggling students in groups such that they be supported by more advanced students. Otherwise, she has learned, these struggling students disengage when working on their own. At times, the teacher intervenes in group work so as to enable more learning opportunities, and in doing so she attempts to detect and imitate emergent properties of student collaboration. For example, regarding the "implementers" and how that role arose, the teacher said,

> "that's what ended up happening [] even within the groups. The <u>kids kind of figured it out</u> themselves, but I think <u>sometimes I prompted them</u>, too. If a kid was just sitting there totally lost, then I know there were times I made that suggestion. The other kids – there were two or three who were on the same wavelength, setting a plan, I said, "Ok, just dot it—let them be the colorer." So <u>they were kind of organizing jobs</u>, you know."

Collaborative projects that frame learning processes. The teacher evaluated that the combinations-tower project benefited about 80-to-85% of the students. Regarding the remaining 15-to-20% of her classroom, she observed that these are students who are not engaged in traditional/frontal teaching, either. The teacher was surprised and gratified by students' learning in this project, as she interpreted the post-test results. These results, she said, are testimony that students learn not only by doing but also by interacting with other students who are more advanced than they are. Accordingly, the teacher pointed to the immersive quality of the construction project as conducive to collaborative learning—it created space, time, context, structure, and opportunities for group interaction and, thus, for tacit apprenticeship and mentorship. The combinations tower was a product that informed a process—the product was important to focus students, sustain their motivation, organize and coordinate their efforts coherently, and ultimately a source of satisfaction and pride, yet the learning was in the process.

When students are given the freedom to explore a problem collaboratively, both remarkable and undesirable group behaviors may emerge. It is not a zero-sum game—these "pros and woes" need not cancel each other out. An experienced and able teacher who anticipates this emergence and is sensitive to unforeseen behavior can steer and ride this sensitive dependence so as to maximize student sharing and learning opportunities (see also Senge & Käufer, 2000, on the *leadership paradox*).

Finally, we wish to emphasize that through analyzing the data we have only grown to respect the teacher more. It would be unfair to state that her good intentions went awry. The teacher was facilitating a new unit involving complicated mathematical reasoning and challenging logistics and management. She organized students in an attempt to optimize their learning opportunities. We are using the classroom data not to judge her practice but to reveal nuances of socio-mathematical phenomena.

Implications of the Study

The results of this study, and in particular our demonstration of student distribution by project role, go beyond issues of design, teaching, and learning—they inform issues of equity and student identity, as we now explain. The study was conducted in a school district that is segregated and has been working hard to close achievement gaps. Table 3 (see below) shows student distribution in our study G6 classrooms by socio-economical status (SES) and by mathematical achievement. Almost all high-SES students are in the top-third mathematical achievement group of their classroom, and no high-SES student is in the bottom third. Two-thirds of the low-SES students are in the bottom-third mathematical achievement group, and no low-SES student is in the top third. All students in the A1 cell are White; all students in the C3 cell are either African–American or Hispanic. Within a classroom population where a student's SES and/or ethnicity predict their mathematical achievement, a student's role in a collaborative project takes on more baggage than in classrooms where student background factors and mathematical achievement are not related. Imagine the scene: in Groups #1 there are 7 students of color, while in Group #4 (morning) and #3 (afternoon) there is just 1 student of color. Amongst the coordinators, number crunchers, and designers there was not a single minority student.

Table 3.
Student Distribution by Mathematical Achievement Group and Socio-Economical Status

Mathematical Achievement Group	SES		
	A*	B	C
1	8	3	-
2	3	7	5
3	-	2	10

*"A" is the highest-SES third

The teacher's premise, based on her observations of student behavior in classroom projects, was that group-work activity designs foster collaborative learning, including spontaneous inter-student mentoring, vicarious learning of peripheral participants, and generally more on-task involving of more students. Yet, given students' emergent distribution by mathematical role, and given the SES/ethnic correlates of this distribution, we submit that more sensitivity to social implications of activity design is required so as to help re-position students as equal participants. That is, designers of collaborative construction projects, in which classroom natural social dynamics are tapped, should study, address, and possibly leverage the emergent social consequences of these designs. Otherwise, the social dynamics may engender sour social consequences. Optimal design should create mathematical havens that foster alternatives to students' extramural givens, such as their SES and its identity baggage. Throughout this paper, we have intentionally spoken of student mathematical *achievement*, because we believe that students have more mathematical *capacity* than is being expressed in terms of their achievement, and this belief informs our search for design frameworks that are equally demanding and equally supportive of all students.

We propose the term *stratified learning zone* to depict a design-driven non-continuousness of students' potential learning trajectories along problem-solving skill sets.[10] In comparison, the term *continuous learning zone* depicts a space wherein students can each embark from a core problem, sustain engagement in working on this problem, and build a set of skills wherein each accomplishment suggests, contextualizes, and supports the exploration and learning of the successive skill, so that a solution path is learned as a meaningful continuum. Finally, When students are all tackling the same core problem, they are more likely to understand, appropriate, and apply with understanding their classmates' problem solving.

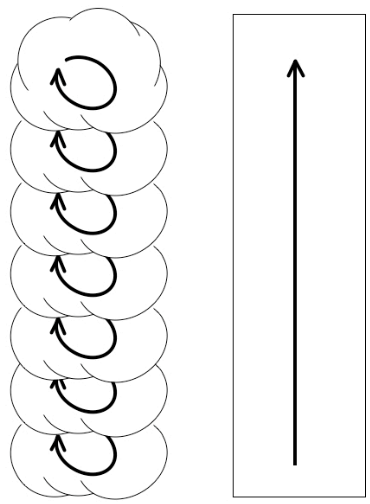


Figure 7. The stratified learning zone (on left) and the continuous learning zone.

In Figure 7 (above, on the left), the stratified learning zone, each stratum represents a problem-solving skill students learn, beginning from the bottom, and each curved arrow represents students practicing that skill. Note that these arrows each remain within their respective strata. A liability of collaborative design is that the stronger students gravitate down (to the lower strata)—they focus on the more fundamental aspects of problem solving, which appeal to them, and relegate the other strata to not-as-strong students, who repeat the relegation recursively. Such dynamics are absent in the case of the *continuous learning zone*. Yet what are design principles for collaborative project-based learning that eschew the stratification of the learning zone?

The activity design, *collaborative* construction, creates conditions that enable spontaneous fragmentation of the overall task. What, during the pre-grouping stage, had been individual continuous paths—concatenations of quasi-discrete analysis, trial-and-error experimentation, and through to coloring in—brakes apart into segments (see Figure 8, below). So, in principle, one student could focus on the first segment (analysis only), another on the second (implementing a design), and yet another on the third (production), etc. Such fragmented activity is liable to foster

[10] By "skill," we mean, simply, the ability to do something. More formally, a skill is a goal-oriented, loosely stable, and repeatable coordinated set of interactions, possibly in multiple modalities and media that constrain and sequence the interactions. A skill may be "devising a method to analyze a combinatorial space of a new stochastic object" or "cutting and pasting paper squares."

student *epistemological anxiety* (Wilensky, 1997), a predicament by which students know *that* their mathematical practice is correct but do not have the mathematical wherewithal to make sense of *why* the practice is correct—they had never had personal opportunities to connect all the way through from initial insight to the solution of a problem.

CLZ \ SLZ	Ann	Bert	Cathy	David
Ann	Skill 1	Skill 2	Skill 3	Skill 4
Bert	Skill 1	Skill 2	Skill 3	Skill 4
Cathy	Skill 1	Skill 2	Skill 3	Skill 4
David	Skill 1	Skill 2	Skill 3	Skill 4

Figure 8. Learning trajectories in continuous (CLZ) and stratified (SLZ) learning zones.

An analogy to a different mathematical learning goal may help here. Consider the case of word problems that students model and solve using algebraic equations. In a continuous learning zone (see Figure 8, above, from left to right), students would progress, possibly along different learning paths, towards developing skills necessary for solving a system of two equations with two unknowns. It would be unhelpful if one student were ultimately only capable of modeling the problem in the form of equations, another student could only perform operations on both sides of an equation, yet another student could only use substitution, and finally another student could only interpret the values determined for the variables in terms of the problem context. Even if a classroom collaboratively invented the algebraic solution, we would still wish for all students to be able to complete the entire solution individually. The irony is that in a stratified learning zone, the weaker a student, the further along the solution path is his or her role, so that the weakest student may engage in performing the finishing touches on a mathematical solution task s/he doesn't know how to begin. Moreover, in a stratified learning zone, a student might find herself in a group where she is tasked with implementing a design at a level of complexity that she could have tackled successfully if only she had had ample opportunity to practice such implementation on a simpler exemplar. But is the stratified learning zone a necessary evil of collaborative-learning designs?

Constructionist design broadens the classroom's learning zone, enabling more students to engage at their level and pace (Kafai & Harel, 1991; Abrahamson, Blikstein, Lamberty, & Wilensky, 2005). A premise of design for *collaborative* learning is that it is a more authentic practice; that outside of the classroom, in both work and play, people commonly have opportunities to negotiate, pool, and coordinate their resources, such as in problem solving (see Lave & Wenger, 1991). From this perspective, *individual* learning is seen as the anomaly. Yet, certain designs for collaborative work may inhibit student progress—sometimes good teamwork does not amount to good learning. A distribution of expertise is desirable in workplaces but it is problematic in mathematics classrooms. Ultimately, students will walk out of schools as individuals and not as a group, so all students should have mastered the content being learned. Durkheim (1947) analyzed societies as held together through mutual dependencies, with individuals gradually growing into the identities prescribed by their individual circumstances and labor-related specialization, a process that Durkheim associated with regression in the collective consciousness. A design that breaks away from traditional classroom management-and-pedagogy practices and taps into students' social motivations must accommodate and wield these powers sensitively. It is not enough to give a classroom tools and ask them to build. It is not enough to distribute the work between students according to their abilities. The moment you unleash a classroom, both the pros and the woes of collaboration snap back.

Given the classroom stratification that emerges when students are completely free to choose their role in a collaborative project, and given the limitations of the learning experiences that such complete freedom may engender, an ostensible solution may bee to introduce constraints into the design so as to enhance student experience of multiple roles and potentially foster "cross-pollination" between task-specific groups (e.g., "jigsaw" design, Aronson, Blaney, Srephan, Sikes, & Snapp, 1978). However, we find these designs wanting. The built-in constraints introduced by a design for collaborative learning should appear to students as necessary and grounded in and motivated by the content, and not as arbitrary role switching. Otherwise, the design is liable to undermine students' sense of ownership over their artifacts. In contrast, a well-tuned collaborative-construction activity may: (a) afford opportunities for authentic peer-to-peer mentorship that engender "mathematical upward-mobility" in demographically-diverse classrooms—with more advanced students explaining to their less advanced classmates not only the mathematical practices but also the logic behind them; (b) foster positive socio-mathematical classroom norms of goal-based inquiry and meaning-making; (c) create classroom experiences that are sufficiently shared so as to ground 'mathematization' but diverse enough so as to stoke constructive debate; (d) allow for all students to contribute in creating a shared artifact and to develop local expertise as well as for advanced students to assume leadership; and (e) develop mature teamwork skills that include co-education and not exploitation.

We wish to point to a design framework that we believe holds promise in supporting equitable facilitation of collaborative learning and thus responds to the design problem we have discussed. In a complementary paper (Abrahamson & Wilensky, 2005d) we describe an implementation of a networked-classroom participatory simulation activity in the same Grade 6 classroom that participated in this study. Classroom network technologies (Roschelle, Penuel, & L. Abrahamson, 2004; Ares et al., 2004) have been shown to enhance student participation. Such technologies, such as the HubNet participatory-simulation architecture (Wilensky & Stroup, 1999b), have demonstrated the potential of networked learning environments to draw students into interactive experimentation (Abrahamson & Wilensky, 2004b). Students all either engage in the same roles or move fluently between roles, yet each student brings individual reasoning to bear as the entire group engages in collaborative problem solving (see, for example, Berland & Wilensky, 2004). Because all student actions are exposed to the classroom milieu (even though these actions may be anonymous), and because students are all engaged in essentially the same tasks, often with shared objectives, there are many opportunities in participatory-simulation activities for constructive feedback from more advanced classmates. These opportunities are more numerous and varied as compared to designs in which student roles create task-specific isolated groups with less mentoring opportunities. Thus, participatory-simulation designs encourage inclusion, sharing, diverse strategies, and peer mentorship, towards achieving more equitable classrooms.

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Appendix A—The "Anchor–Mover" Algorithm for Producing All Possible Different 9-blocks That Each Have Exactly 2 Green Squares

1	2	

1		3

1		
4		

1		
	5	

1		
		6

1		
7		

1		
	8	

1		
		9

	2	3

	2	
4		

	2	
	5	

	2	
		6

	2	
7		

	2	
	8	

	2	
		9

		3
4		

		3
	5	

		3
		6

		3
7		

		3
	8	

		3
		9

4	5	

4		6

4		
7		

4		
	8	

4		
		9

	5	6

	5	
7		

	5	
	8	

	5	
		9

		6
7		

		6
	8	

		6
		9

7	8	

7		9

	8	9